Sign Variable Coulomb Like Potential And The Nonlocal Quantum Theory

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In memory of G.V.Efimov and his 90 years.

***Abstract*—By using the sign variable Coulomb like potential we construct the nonlocal quantum electrodynamics free from ultraviolet divergences.**

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I. INTRODUCTION

During 20 Century main problem of physics was elimination from ultraviolet divergences in quantum physics. Many physicists' works were devoted to this problem and finally, it leads to the Standard model [1]. Among many attempts, the nonlocal quantum field theory was played an important role in the solution of this problem. This approach is more attractive, because in it there exist concept of extended objects and some characteristic length named fundamental length. This scheme leads to the nonlocal interactions of quantized fields. Historically, among these directions the nonlocal theory with a form factor was initiated from Pauli and Villars work [2], recently by Efimov [3] and author of this paper [4].

It is well known that the singularity in classical electrodynamics was cornerstone of the ultraviolet divergence in the quantum electrodynamics. Indeed, singularity of the Coulomb potential at small distances

$$U_c(r) = \frac{e}{4\pi}\frac{1}{r} \qquad (1)$$

is automatically leads to divergence of the photon propagator in the static limit with using the Yukawa corresponding principle:

$$D_c(\vec{p}) = \frac{1}{e}\int d^3r e^{i\vec{p}\vec{r}}\ U_c(r) = \frac{1}{\vec{p}^2} \qquad (2)$$

or in x – space:

$$D_c^{\mu\nu}(x) = \frac{1}{(2\pi)^4 i} g^{\mu\nu} \times \int d^4p\ e^{ipx} \frac{1}{p_0^2 - \vec{p}^2 + i\varepsilon}. \qquad (3)$$

To eliminate this divergence, we introduced following finite potential form:

$$U_c^\ell(r) = \frac{e}{4\pi}\frac{1}{\sqrt{r^2+\ell^2}} \qquad (4)$$

and constructed the finite nonlocal quantum electrodynamics [5]. Also it is shown that if it is introduced the potential of the stick or dipole of the form

$$V_d(r) = \frac{1}{4\pi}\frac{e}{r}\left(1+\frac{2\ell}{r}\right) \qquad (5)$$

leading to the appearance of propagators of the photon and photino all together:

$$D_{\mu\nu}^{e\gamma}(x) = \frac{1}{(2\pi)^4 i} g_{\mu\nu} \int d^4p\ e^{ipx} \times \left[\frac{1}{p^2+i\varepsilon} + \pi\ell\frac{(-i\hat{p})}{p^2+i\varepsilon}\right],$$

$$p^2 = p_0^2 - \vec{p}^2, \qquad (6)$$

then one can construct a finite theory free from ultraviolet divergences [6].

Worth notice that these two schemes (4) and (5) allow us to calculate nonlocal and photino constributions to the muon anomalous magnetic moment (AMM):

$$(\Delta\mu)_{nonlocal} = \frac{\alpha}{2\pi}\left[1 + \frac{m_\mu^2\ell^2}{6}\left(ln\frac{m_\mu^2\ell^2}{4} + \frac{1}{6} - 2\psi(1)\right)\right] \qquad (7)$$

and

$$(\Delta\mu)_{photino} = \frac{4}{15}\alpha m_\mu \ell \qquad (8)$$

By using experimental data [7] and theoretical calculations [8] for the muon AMM one can obtain from these two expressions (7) and (8) corresponding characteristic lengths [9]:

$$\ell_{non} = 1.93 \times 10^{-15} m \qquad (9)$$

and

$$\ell_{phot} = 1.28 \times 10^{-25} m \qquad (10)$$

respectively. It means that the difference between $a_\mu^{exp} - a_\mu^{sm}$ is explained by means of the nonlocal theory and an existence of the photino.

II. Sign Variable Potentials

a. Concrete Forms of Potentials

We think that sign variable potentials play an important role in creation of bound states in physics. For example, two electrons due to these potentials in the Coulomb interactions may be form bound states to get Cooper pairs, i.e., boson condensations. This phenomenon plays a vital role in superconductivity and superstream and so on.

Let us consider the following sign variable Coulomb type potential

$$U_s^1(r) = \frac{e}{4\pi}\frac{1}{r} J_0\left(\sqrt{r/\ell}\right), \qquad (11)$$

where $J_0(x)$ is the Bessel function of the zero-order:

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(1+n)}\left(\frac{x}{2}\right)^{2n} = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{1}{sin\pi\xi\,\Gamma^2(1+\xi)}\left(\frac{x}{2}\right)^{2\xi}, \quad (-1<\beta<0), \qquad (12)$$

This is the Mellin representation for $J_0(x)$ – function.

Notice that there exists another complicated sign variable potential:

$$U_s^2(r) = \frac{e}{2\pi}\frac{1}{r} J_0\left(\sqrt{r/\ell}\right) K_0\left(\sqrt{r/\ell}\right), \qquad (13)$$

where $K_0(x)$ is the Macdonald function of the zero order:

$$K_0(x) = -I_0(x) ln\frac{x}{2} + \sum_{k=0}^{\infty} \frac{x^{2k}}{2^{2k}(k!)^2}\psi(1+k). \qquad (14)$$

Here

$$I_0(x) = \sum_{n=0}^{\infty} \frac{1}{(n!)^2}\left(\frac{x}{2}\right)^{2n}. \qquad (15)$$

b. Nonlocal Photon Propagator

In the static limit, Fourier transform of potentials (11) and (13) gives corresponding nonlocal photon propagators:

$$D_1(\vec{p}) = \frac{1}{\vec{p}^2} cos\left(\frac{1}{4|\vec{p}|\ell}\right), \qquad (16)$$

and

$$D_2(\vec{p}) = \frac{1}{\vec{p}^2} K_0\left(\frac{1}{2|\vec{p}|\ell}\right). \qquad (17)$$

In this paper, we use only propagator (16) which is given by in the four-dimensional case:

$$D_{\mu\nu}^1(x) = \frac{1}{(2\pi)^4}\frac{1}{i} g_{\mu\nu} \int d^4p\, e^{ipx} \times cos\left(\frac{1}{4\ell\sqrt{-p^2}}\right)\frac{1}{p^2+i\varepsilon}, \qquad (18)$$

where

$$cos\left(\frac{1}{4\ell\sqrt{-p^2}}\right) = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \times \frac{\left(\frac{1}{4}\right)^{2\xi}}{sin\pi\xi\,\Gamma(1+2\xi)}\left(\frac{1}{-p^2\ell^2}\right)^{\xi} \qquad (19)$$

$$p^2 = p_0^2 - \vec{p}^2, \qquad -1<\beta<0.$$

For concrete calculational purpose, the formula (18) can be write in convenient form:

$$D_{\mu\nu}(x) = D_{\mu\nu}^1(x) = \frac{1}{(2\pi)^4}\frac{1}{i} g_{\mu\nu} \times \int d^4p\, e^{ipx}\frac{V(-p^2\ell^2)}{p^2+i\varepsilon}, \qquad (20)$$

Here

$$V(-p^2\ell^2) = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{v(\xi)}{sin\pi\xi}(-p^2\ell^2)^{-\xi}, \qquad (21)$$

$$v(\xi) = \frac{1}{\Gamma(1+2\xi)} 4^{-2\xi} \qquad (22)$$

c. The Interaction Lagrangian and S-Matrix

The interaction Lagrangian of the nonlocal quantum electrodynamics arisen from the modification of the Coulomb potential (11) has similar structure as the local theory [10].

$$L_{in}(x) = e{:}\bar{\psi}(x)\hat{A}(\ell,x)\psi(x){:} \qquad (23)$$

and corresponding S-matrix takes the form

$$S = Texp\left\{i\int d^4x\, L_{in}(x)\right\} \qquad (24)$$

Here

$$\hat{A}(\ell,x) = A_\mu(\ell,x)\gamma^\mu.$$

Here "chronological" pairing (or T-product) of the fermonic field operators of electrons has the usual local form:

$$S(x-y) = \langle 0|T\{\bar{\psi}(x)\psi(y)\}|0\rangle =$$

$$\frac{1}{(2\pi)^4}\frac{1}{i}\int d^4p\,\frac{e^{-ip(x-y)}}{m-\hat{p}-i\varepsilon} \qquad (25)$$

while causal Green function of the nonlocal electromagnetic field $A_\mu(\ell,x)$ in (23) acquires the form

$$D_{\mu\nu}(x-y) = \langle 0|T\{A_\mu(\ell,x)A_\nu(\ell,y)\}|0\rangle =$$

$$g_{\mu\nu}D^\ell(x-y) =$$

$$\frac{1}{(2\pi)^4}\frac{g_{\mu\nu}}{i}\int d^4p\,\frac{V(-p^2\ell^2)}{p^2+i\varepsilon}e^{-ip(x-y)} \qquad (26)$$

due to formula (20).

Further. as usual in our case Feynman diagrams with the exception of closed fermion loops are investigated all which are finite at high momentum.

d. Vacuum Polarization

In our theory the vacuum polarization does not changed and has the standard structure in x-space:

$$\Pi^{\mu\nu}(x-y) =$$

$$-ie^2Tr\{\gamma^\mu S(x-z)\gamma^\nu S(z-y)\}. \qquad (27)$$

In the momentum space, after some traditional calculations, we have

$$\Pi(q^2)\frac{e^2}{2\pi^2}\int_0^1 dx\,x\,(1-x)\times$$

$$ln\left(1+\frac{q^2x(1-x)}{m^2}\right). \qquad (28)$$

Here

$$\Pi^{\mu\nu}(q) = (g^{\mu\nu}q^2 - q^\mu q^\nu)\Pi(q^2) \qquad (29)$$

e. Lepton Self-Energy

In this work, we do not draw primitive Feynman diagrams which were done in my many papers [5,6,9]. Thus in this nonlocal theory the self-energy has the form

$$\Sigma_\ell\,(x-y) =$$

$$-ie^2\gamma_\mu S(x-y)\gamma_\mu D^\ell(x-y) \qquad (30)$$

After some calculations, we have in the momentum space:

$$\tilde{\Sigma}_\ell(p) = \frac{e^2}{8\pi}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi\,\frac{1}{sin^2\pi\xi}\times$$

$$\frac{v(\xi)(m^2\ell^2)^{-\xi}}{\Gamma(1-\xi)}F(\xi,p) \qquad (31)$$

where

$$v(\xi) = \frac{1}{\Gamma(1+2\xi)}4^{-2\xi}$$

and

$$F(\xi,p) = \frac{1}{\Gamma(1+\xi)}\int_0^1 du\left(\frac{1-u}{u}\right)^{-\xi}\times$$

$$\left(1+\frac{p^2}{m^2}u\right)^{-\xi}(2m-\hat{p}u) \qquad (32)$$

f. The Vertex Function

In the momentum space and in the Euclidean metric, the vertex function takes the form

$$\tilde{\Gamma}^\ell_\mu(p_1,p) = -\frac{e^2}{(2\pi)^4}\int d^4k_E\,\frac{V((p_E-k_E)^2\ell^2)}{(p_E-k_E)^2}\gamma_\nu\times$$

$$\frac{m-\hat{k}_E-\hat{p}_E}{m^2+(k_E+p_E)^2}\gamma_\mu\frac{m-\hat{k}_E}{m^2+k_E^2}\gamma_\nu. \qquad (33)$$

After some standard calculations, we have

$$\tilde{\Gamma}^\ell_\mu(p_1,p) = -\frac{e^2}{8\pi}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi\,\frac{v(\xi)}{sin^2\pi\xi}\times$$

$$\frac{(m^2\ell^2)^{-\xi}}{\Gamma(1+\xi)}F(\xi,p_1,p) \qquad (34)$$

where

$$\mathrm{F}_\mu(\xi;p_1,p) = \gamma_\mu\mathrm{F}_1(\xi;p_1,p) + \mathrm{F}_{2\mu}(\xi;p_1,p)$$

Here

$$\mathrm{F}_1(\xi;p_1,p) = \frac{1}{\Gamma(1+\xi)}\int_0^1\int_0^1\int_0^1 d\alpha d\beta d\gamma$$

$$\times\ \delta(1-\alpha-\beta-\gamma)\alpha^\xi Q^{-\xi}, \qquad (35)$$

$$\mathrm{F}_{2\mu}(\xi,p_1,p) = \frac{1}{\Gamma(\xi)}\int_0^1\int_0^1\int_0^1 d\alpha d\beta d\gamma\times$$

$$\delta(1-\alpha-\beta-\gamma)\times$$
$$\alpha^{\xi}Q^{-1-\xi}\frac{1}{m^2}\Big[m^2\gamma_\mu-2mq_\mu+4m(\beta q_\mu-\alpha p_\mu)+$$
$$+(\alpha\hat{p}-\beta\hat{q})\gamma_\mu\hat{q}+(\alpha\hat{p}-\beta\hat{q})\gamma_\mu(\alpha\hat{p}-\beta\hat{q})\Big] \qquad (36)$$

and

$$Q=\beta+\gamma-\alpha\gamma\frac{p^2}{m^2}-$$
$$-\beta\gamma\frac{q^2}{m^2}-\alpha\beta\frac{(p+q)^2}{m^2}.$$

By using expressions (34)-(36) one can obtain detail results as before [5,6,9].

In this approach it is interesting to calculate contribution to the lepton anomalous magnetic moment

$$(\Delta\mu)_{lepton}=\frac{\alpha}{2\pi}\left[1+\frac{2}{3}\upsilon(-1)m^2\ell^2\right]. \qquad (37)$$

Notice that to obtain this formula we can move the counter integration to the left in above formulas. For the anomalous MM for the muon, the difference between the experimental data and theoretical calculations acquires the form [7,8]:

$$\Delta=a_\mu^{exp}-a_\mu^{SM}=(25.1\pm5.9)\times10^{-10}. \qquad (38)$$

It turns out that the nonlocal theory with the form factor (22) does not give contribution to this difference, because

$$\upsilon(-1)=\frac{16}{\Gamma(-1)}=0.$$

It means that charged leptons are trapped at rest in the origin of the coordinate system.

g. Appendix 1

Now let us calculate the photon propagator arisen from the sign variable potential (13).

For this purpose, we use the integral representation for $K_0(z)$ - function [11]:

$$K_0(\ell z)=\int_0^\infty dt\frac{cos\ell t}{\sqrt{t^2+z^2}} \qquad (39)$$

where

$$cos\ell t=\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\xi\frac{(\ell t)^{2\xi}}{sin\pi\xi\Gamma(1+2\xi)}. \qquad (40)$$

Thus

$$\int_0^\infty dt\frac{t^{2\xi+1-1}}{(z^2+t^2)^{1/2}}=\frac{1}{2z}(z^2)^{\frac{2\xi+1}{2}}\times$$
$$\frac{\Gamma\left(\frac{2\xi+1}{2}\right)\Gamma\left(\frac{1}{2}-\frac{1+2\xi}{2}\right)}{\Gamma\left(\frac{1}{2}\right)}.$$

Here

$$\Gamma(-\xi)\Gamma(1+\xi)=-\frac{\pi}{sin\pi\xi}.$$

Therefore

$$K_0(\ell z)=-\frac{\sqrt{\pi}}{4i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\xi\ \times$$
$$\frac{(z\ell)^{2\xi}}{sin^2\pi\xi}\frac{\Gamma\left(\frac{1}{2}+\xi\right)}{\Gamma(1+2\xi)\Gamma(1+\xi)}. \qquad (41)$$

Then relativistic extension of (17) gives the nonlocal photon propagator arisen from the potential (13) an explicit form of which acquires the form:

$$D_{\mu\nu}^2(x)=\frac{1}{(2\pi)^4}\frac{1}{i}g_{\mu\nu}\int d^4p\ e^{ipx}\times$$
$$\frac{V_2(-p^2\ell^2)}{p^2+i\varepsilon} \qquad (42)$$

where

$$V_2(-p^2\ell^2)=\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\xi\frac{\upsilon_2(\xi)}{sin\pi\xi}(-p^2\ell^2)^{-\xi}. \qquad (43)$$

Here

$$\upsilon_2(\xi)=-\frac{\sqrt{\pi}}{2}\times$$
$$\frac{\Gamma\left(\frac{1}{2}+\xi\right)}{sin\pi\xi\Gamma(1+\xi)\Gamma(1+2\xi)}2^{-2\xi} \qquad (44)$$

Notice that construction of the nonlocal quantum electrodynamics with using the photon propagator (42) encounters some calculational difficulties. This approach with the photon propagator (42) is very similar to our previous work [5].

In conclusion it is notice that sign variable potential approach with the formfactors (21) and (43) which respect to the nonlocal theories arisen from the Coulomb like potentials (4) and (5) give an essential nonlocal theory, where at the limit $\ell \to 0$ it does not going to the usual local theory.

h. Appendix 2

Photino potentials, propagators and their nonlocal formfactors

In our scheme, the Coulomb like potential for photino takes the form:

$$U_{phot}(r) = \frac{e}{2\pi^2}\frac{1}{r^2}. \qquad (45)$$

Therefore the photino propagator in the static limit acquires the standard form:

$$D_{phot}(|\vec{p}|) = \frac{1}{e}\frac{1}{2\pi^2} \times \int d^3r\, e^{i\vec{p}\vec{r}}\frac{1}{r^2} = \frac{1}{|\vec{p}|} \qquad (46)$$

or the relativistic extension gives

$$D_{phot}(p) = -i\frac{\hat{p}}{p^2 + i\varepsilon}, \quad \hat{p} = \gamma^\nu p_\nu \qquad (47)$$

Now let us obtain a nonlocal formfactors for photino. For this purpose, one can consider the following potential form

$$U_{phot}^{non}(r) = \frac{e}{2\pi^2}\frac{1}{r^2 + \ell^2} \qquad (48)$$

The Fourier transform of this potential in the relativistic case gives

$$D_{phot}^{\ell}(p) = \frac{-i\hat{p}}{p^2 + i\varepsilon}e^{-\ell\sqrt{-p^2}}, \qquad (49)$$

where for this formfactor the following Mellin representation is valid

$$e^{-\ell\sqrt{-p^2}} = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \times \frac{1}{\sin\pi\xi\Gamma(1+\xi)}\ell^{\xi}(-p^2)^{\xi/2} \qquad (50)$$

Notice that there exist many possibilities to obtain nonlocal photino propagators. Here we give some examples. Let

$$U_{phot}^{2n}(r) = \frac{1}{r^2 - \ell^2}\cdot\frac{e}{2\pi^2} \qquad (51)$$

Then, we have

$$D_{phot}^{2\ell}(p) = \frac{-i\hat{p}}{p^2 + i\varepsilon}\cos\left(\ell\sqrt{-p^2}\right), \qquad (52)$$

$$\cos\left(\ell\sqrt{-p^2}\right) = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \times \frac{1}{\sin\pi\xi\Gamma(1+2\xi)}\ell^{2\xi}(-p^2)^{\xi} \qquad (53)$$

Further, let us consider the following potential form

$$U_{phot}^{3n}(r) = \frac{e}{2\pi^2}\frac{r^2}{r^4 + \ell^4}$$

then we have

$$D_{phot}^{3\ell}(p) = \frac{-i\hat{p}}{p^2 + i\varepsilon}\exp\left(-\frac{\ell\sqrt{-p^2}}{\sqrt{2}}\right) \times \cos\left(\frac{\ell\sqrt{-p^2}}{\sqrt{2}}\right), \quad p^2 = p_0^2 - \vec{p}^2 \qquad (54)$$

This is a complicated formfactor.

Let us consider the potential form for the photino:

$$U_{phot}^{4n}(r) = \frac{e}{\pi^2}\frac{r^2}{(r^4 - \ell^4)} \qquad (55)$$

This form of the potential gives

$$D_{phot}^{4\ell}(p) = \frac{-i\hat{p}}{p^2 + i\varepsilon} \times \left[e^{-\ell\sqrt{-p^2}} + \cos\left(\ell\sqrt{-p^2}\right)\right] \qquad (56)$$

Finally, we use the following potential form

$$U_{phot}^{5n}(r) = \frac{e}{2\pi^2}\frac{I_0^{-1}(1)}{r^2 + \ell^2}J_0\left(\frac{r}{\ell}\right), \qquad (57)$$

where

$$I_0(1) = \sum_{n=0}^{\infty}\frac{1}{(n!)^2}\left(\frac{1}{2}\right)^{2n}$$

is the cylinder function. Then we have

$$D_{phot}^{5\ell}(p) = \frac{-i\hat{p}}{p^2 + i\varepsilon}e^{-\ell\sqrt{-p^2}} \qquad (58)$$

The above mentioned cases mean that we can construct the photino nonlocal theory.

The photino theory with above mention formfactors is free from the ultraviolet divergences. Here the parameter ℓ is an order of $\ell \leq 2 \times 10^{-25}m$.

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