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Experimental And Matlab-Aided Determination Of The Parameters Of A Transformer And Its Performance Under Magnetic Saturation

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Abstract—* *The operational parameters of a refurbished 3000VA single-phase inverter transformer were investigated. The use of MATLAB was necessary to facilitate the exercise. In this paper, the author gives details of the significant transformer performance equations and by application of MATLAB software. The results of the no-load and short-circuit laboratory experiments, the necessary MATLAB programs were written. On test-running the programs, six major transformer equivalent circuit parameters were obtained as a precursor to the determination of the transformer performance data. By the same round of test-running, the refurbished transformer yielded the following performance details: no-load exciting current = 4.10A; core losses = 33.90W; copper losses = 157.18W; efficiency = 89.74%; and voltage regulation = 10.35%, to mention key parameters. The magnetization curve of the apparatus was also obtained in the process. And by means of the latter, the saturation level, K_{sat}, of the transformer was manually estimated and the final round of test-running the MATLAB program was undertaken. With the K_{sat} value of 3.0 p.u., the above main performance data became: no-load current = 12.09A; core losses = 38.55W; copper losses = 239.75W; efficiency = 89.57%; and voltage regulation = 9.38%. And the inverter transformer was adjudged to be fairly okay.

Keywords—* *Transformer, Parameters, Performance, Saturation, Matlab.

I. INTRODUCTION

The transformer is the heartbeat of any electrical equipment. The use of transformers in both high-voltage power system networks and low-voltage equipment is inevitable [1]. The transformer used in this research is a refurbished 3000VA, 230/48V, 50Hz single-phase inverter transformer. It became defective due to a serious overload when it was made to charge a drained inverter battery bank. The normal load, however, involved mainly illuminating lamps and office/home cooling fans. The estimated input power factor, considered to accommodate the actual load power factor, was, therefore, 0.9 p.u., lagging [1]. After refurbishment, subjecting it to a steady-state computer-aided performance evaluation became necessary.

An investigation carried out without saturation (and or skin effect, harmonic effect, etc.) amounts to a linearization approach to avoid cumbersome modeling and computations. Including saturation, as in this survey, introduces non-linearity and so brings the researcher closer to the real performance of the equipment. To this end, the open-circuit characteristic (OCC) of the transformer is vital and is usually obtainable from the no-load test of the apparatus, other tests notwithstanding.

Thus, the work here begins with the production of the relevant transformer performance equations and the presentation of the laboratory no-load and short-circuit experimentation results all which are featured in Section 2.0. In Section 3.0, the author gives details of the results of the test-running of the computer programs generated; whilst, in Section 4.0 discussion, conclusion and recommendation together brings up the rear.

II. PERFORMANCE EQUATIONS

The ***exact equivalent circuit*** for a transformer is as shown in Fig.1. Sometimes, when exact computational results are not required the core loss and magnetizing branch is moved to the supply terminals, as shown in Fig.2. The error introduced by this does not usually exceed 2 – 3% [2].

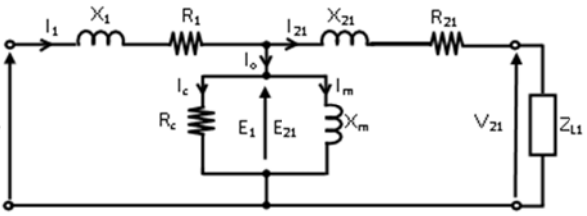

Fig.1: Exact T-Equivalent Circuit of any Transformer with the secondary parameters referred to the primary.

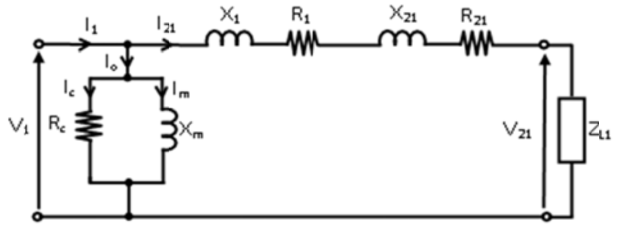

Fig.2: Approximate Equivalent Circuit of a Small-Sized Transformer with the secondary parameters referred to the primary.

Also, since generally the no-load current, I_o, is hardly 3 to 5% of the full-load rated current, this parallel branch consisting of the resistance R_c and reactance X_m can be omitted completely without introducing any appreciable error in the behaviour of the transformer under loaded condition [3], especially in larger transformers [4].

The equivalent circuit parameters are defined as follows: V_1 – the applied terminal phase voltage to the primary winding; E_1 – the back emf produced in the primary winding due to V_1; E_{21} – the induced secondary emf referred to the primary; I_1 – the phase current in the primary winding driven by V_1; I_{21} – the load component of the primary current, designated as the secondary phase current referred to the primary; I_o – the core exciting (or no-load) current; I_c – The power loss component of the core exciting current; I_m – the core magnetizing current component of the exciting (or no-load) current (this current is responsible for the setting up of the mutual flux, ϕ_m, in the apparatus and is therefore associated with the hysteresis loss); R_1 – the series per-phase primary winding resistance, associated with the copper (I^2R) losses in the primary; R_{21} – The series per-phase secondary resistance referred to the primary; R_c – the shunt core (i.e. iron) resistance associated with the iron losses of the apparatus (it is indeed the reflection of the core or magnetic circuit conductance, i.e. $g = 1/R_c$); X_1 – the series per-phase leakage reactance produced in the primary circuit owing to the leakage inductance introduced into the circuit by the stray flux which happens to link only the primary winding; X_{21} – the series per-phase secondary leakage reactance referred to the primary; X_m – the shunt core reactance produced by the mutual inductance resulting from the presence of the mutual flux, ϕ_m, in the magnetic circuit (this is the reflection of the susceptance of the magnetic circuit, i.e. $b = 1/X_m$); Z_{L1} – load impedance referred to the primary.

III. EQUATIONS FOR THE EQUIVALENT CIRCUIT PARAMETER COMPUTATIONS

From the no-load test [4],[5]:

$$I_o = \frac{V_1}{V_{oc}} I_{oc}\,;\; P_{oc1} = \left[\frac{V_1}{V_{oc}}\right]^2 P_{oc} \qquad (1)$$

(no-load current & power on full voltage)

$$\cos\varphi_{oc} = \frac{P_{oc1}}{V_1 I_o};\; \varphi_{oc} = \cos^{-1}\left[\frac{P_{oc1}}{V_1 I_o}\right] \qquad (2)$$

(no-load power-factor & p.f. angle)

$$I_c = I_o \cos\varphi_{oc} \;\; and \;\; \mathrm{I_m} = I_o \sin\varphi_{oc} \qquad (3)$$

(no-load loss & magnetizing currents)

$$Z_o = \frac{V_1}{I_o};\; R_c = \frac{V_1}{I_c};\; and \;\; X_m = \frac{V_1}{\mathrm{I_m}} \qquad (4)$$

(no-load impedance & its components)

$$R_s = R_1 + R_{21} = \frac{P_{sc}}{I_{sc}^{\;2}} \qquad (5)$$

From the short-circuit test:
(transformer circuit series resistance)

$$Z_s = \frac{V_{sc}}{I_{sc}} = R_s + jX_s \qquad (6)$$

(transformer circuit series impedance)

$$X_s = X_1 + X_{21} = \sqrt{\left(Z_s^{\,2} - R_s^{\,2}\right)} \qquad (7)$$

(total transformer circuit series reactance)

$$\theta_s = \tan^{-1}\left(X_s / R_s\right) \qquad (8)$$

(phase angle of the series impedance, Z_s)

$$V_{z(\%)} = \left(V_{sc}/V_1\right) * 100 \qquad (9)$$

(transformer impedance voltage percent)

Temperature-correction of resistances and the associated quantities:

i. *Correction of Resistances and Impedances –* From both tests, the total series resistance, R_s, and the core-loss resistance, R_c, shall be adjusted or corrected in value to that at 75°C as applicable to all electrical apparatus [6], [7]. Let the temperature-corrected versions be designated R_{scc} and R_{cc}, respectively. For the copper-based resistance, we have from [8]:

$$R_{scc} = R_s\left[(234.5+75)/(234.5+T_e)\right] \qquad \text{(10a) and}$$

for the iron or steel based resistance, we shall have

$$R_{cc} = R_c\left[1 + \left\{T_{cr}\left(75 - T_e\right)\right\}\right] \qquad \text{(10b)}$$

where T_e is the equipment temperature at the time of experiment and T_{cr} is the temperature coefficient of resistance of steel which is $0.0045\mathrm{K}^{-1}$ [9].

ii. *Correction of No-Load Circuit Quantities –*

The no-load admittance, impedance, phase angle, power & current become

$$Y_{occ} = \left(\frac{1}{R_{cc}}\right) - j\left(\frac{1}{X_m}\right);\ Y_{occ} = \left[\left(\frac{1}{R_{cc}}\right)^2 + \left(\frac{1}{X_m}\right)^2\right]^{½} \qquad (11a)$$

$$Z_{occ} = \frac{1}{Y_{occ}=} \frac{1}{[Y_{occ}} \angle(-\theta_{occ})]\ and\ \theta_{occ} = tan^{-1}(R_{cc}/X_m) \qquad (11b)$$

$$P_{occ} = [V_1]^2 / R_{cc}\ \ and\ \ \cos\phi_{occ} = P_{occ}/(V_1 I_{occ}) = pf\_oc \qquad (12)$$

$$I_{occ} = \frac{V_1}{Z_{occ}};$$

$$I_{cc} = I_{occ}\cos\phi_{occ}\ \ and\ \ I_{mc} = I_{occ}\sin\phi_{occ} \qquad (13)$$

iii. *Correction of the Relevant Short-Circuit Equivalent Circuit Quantities*

The short-circuit impedance, phase angle, power & currents become

$$Z_{scc} = \sqrt{\left(R_{scc}^{\ 2} + X_s^{\ 2}\right)};$$

$$\theta_{scc} = tan^{-1}(X_s/R_{scc});\ Z_{scc} = Z_{scc}\angle\theta_{scc} \qquad (14)$$

$$I_{scc} = \frac{V_{sc}}{Z_{scc}} = \frac{(V_{sc}\angle 0)}{(Z_{scc}\angle\theta_{scc}}) = \frac{[V_{sc}}{Z_{scc}]\angle(-\theta_{scc}}) \qquad (15)$$

[reduced-voltage short-circuit current]

$$P_{scc} = I_{scc}{}^2 R_{scc};\ pf\_sc = cos\theta_{scc}$$

copper losses & p.f.] (16)

Performance Equations *For Computations without Saturation*

The performance equations shall be obtained from the solution of the approximate equivalent circuit whose phasor diagram is shown in Fig.3 (the primary voltage, V_1, being made the reference quantity).

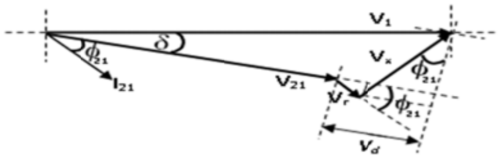

Fig. 3: Phasor Diagram of the Approximate Equivalent Circuit

And with the six (6) major circuit parameters now known, the computations to yield the performance data becomes possible even manually. Of course, the performance of the equipment must be surveyed under load condition, using the no-load and short-circuit temperature-corrected experimental data as and where necessary.

Thus, with the primary voltage taken as the reference quantity (i.e. $V_1\angle 0^o$) and the transformer rated apparent power being $S\angle\phi$, the input current (I_1) can be obtained from the relationship $S_1 = S_1\angle\phi_1 = (V_1\angle 0)I_1^*\ or\ I_1 = \frac{[S_1\angle -\phi_1]}{(V_1\angle 0)} = \frac{(S_1}{V_1})\angle(-\phi_1)$ (17a)

where $\phi_1 = cos^{-1}(pf1)$ and pf1 is the stipulated system input power factor. (17b)

Thus, the load component of the input current and the expected load power factor (pf_{21}) are given as

$$I_{21} = I_1 - I_{occ} = [I_1\angle(-\phi_1)] - [I_{occ}\angle(-\theta_{occ})] = (I_1 cos\phi_1 - I_{occ}cos\theta_{occ}) - j(I_1 sin\phi_1 - I_{occ}sin\theta_{occ})$$

Let $I_{21\_ac} = (I_1 cos\phi_1 - I_{occ}cos\theta_{occ})$ [active component of I_{21}]

and $I_{21\_re} = (I_1 sin\phi_1 - I_{occ}sin\theta_{occ})$ [reactive component of I_{21}]

i.e.

$$I_{21} = \left\{\left[\left(I_{21\_ac}\right)^2 + \left(I_{21\_re}\right)^2\right]^{½}\right\}\angle(-\theta_{21}) = I_{21}\angle(-\theta_{21}) \qquad (18a)$$

$$\phi_{21} = tan^{-1}\frac{[(I_{21\_re})}{(I_{21\_ac})}] \qquad (18b)$$

$pf_{21} = cos\ \phi_{21}$ [expected actual load power factor] (18c)

The transformer impedance voltage drop and associated phase angle are given by

$$V_z = I_{21}Z_{scc} = [I_{21}\angle(-\phi_{21})][Z_{scc}\angle\theta_{scc}] = [I_{21}Z_{scc}]\angle(\theta_{scc} - \phi_{21}) = V_z\angle\theta_z;\ \ \theta_z = (\theta_{scc} - \phi_{21}) \qquad (19)$$

The secondary voltage referred to the primary is expressed as:

$$V_{21} = V_1 - V_z = V_1\angle 0 - V_z\angle\theta_z = (V_1 - V_z cos\theta_z) - jV_z sin\theta_z = \{[(V_1 - V_z cos\theta_z)^2 + (V_z sin\theta_z)^2]^{½}\}\angle\theta_{v21} \qquad (20a)$$

$\theta_{v21} = tan^{-1}\left[\frac{V_z sin\theta_z}{(V_1 - V_z cos\theta_z)}\right]$ [phase angle of V_{21}]

$\delta = \theta_{v21}$ [the displacement or transmission angle] (20b)

The load impedance referred to the primary can be computed as

$$Z_{L1} = \frac{V_{21}}{I_{21}=} \frac{(V_{21}\angle\delta)}{(I_{21}\angle(-\phi_{21})=} \frac{[V_{21}}{I_{21}]\angle(\delta+\phi_{21})} \qquad (21a)$$

and $\theta_{L1} = (\delta + \phi_{21})$ [overall system output impedance angle] (21b)

Total power losses, P_{Loss}, are given as

$P_{Loss} = P_{ir} + P_{co}$; $P_{ir} = P_{occ} = \frac{V_1^2}{R_c}$ and $P_{co} = I_{21}^2 R_{scc}$, respectively (22a)

Active Power deliverable to the Load

$P_{out} = V_{21} I_{21} cos\phi_{21}$ (22b)

Efficiency of the transformer, eff, is then expressed as

$$eff = \left[\frac{P_{out}}{(P_{out}+P_{Loss})}\right] x100 \quad (23a)$$

Maximum efficiency will take place when P_{co} (which depends on load) equals P_{ir} (which is load independent). Hence, we can write

$$eff_{max} = \left[\frac{P_{out}}{(P_{out}+2P_{ir})}\right] x100 \quad (24)$$

Apparent power deliverable by transformer on full load

$S_{out} = V_{21} I_{21}$ (25a)

Apparent power at which maximum efficiency occurs

$$S_{\eta(max)} = S_{out} \left[\left(\frac{P_{ir}}{P_{co}}\right)\right]^{½} \quad (25b)$$

Voltage regulation, where V_1 and V_{21} are known, is simply

$$V_{reg} = \left[\frac{(NoLoad\ to\ FullLoad\ Change\ in\ V_{21})}{(FullLoad\ V_{21})}\right] x10 \quad (26a)$$

In Fig.3 the angle δ is usually very small compared to ϕ_{21}. Hence, to simplify matters further we set δ = 0 and have

$$V_{21} + V_d = V_1 \cos\delta = V_1\ and\ V_1 - V_{21} = V_d = Change\ in\ Secondary\ Voltage \quad (26b)$$

Taking care of both lagging and leading power factor loads the quantity V_d shall be expressed as in [10].

$V_d = I_{21} R \cos\phi_{21} \pm I_{21} X_s \sin\phi_{21}$ (where '+' is for lagging and '–' for leading p.f.)

Therefore

$$V_{reg} \cong \frac{V_d}{V_{21}} = \left\{ I_{21} \frac{[R_{scc} cos\phi_{21} + X_s sin\phi_{21}]}{V_{21}} \right\} x100 \quad (26c)$$

For Computations Including Saturatio

(i) From No-Load Equivalent Circuit and Relevant Experimental Data – The saturation factor, K_{sat}, is as well defined in terms of equivalent circuit reactances as the ratio of $X_{(unsaturated)}$ to $X_{(saturated)}$; where the saturated equivalent circuit reactance, $X_{(saturated)}$, is that obtained from a full-voltage short-circuit test at 75°C according to [7], [11]. That is,

$$K_{sat} = \frac{X_{unsaturated}}{X_{saturated}};\ X_{saturated} = \frac{X_{unsaturated}}{K_{sat}} \quad (27)$$

It is necessary, where saturation is involved, to use the exact equivalent circuit of the electrical apparatus, as in [11]. Saturation has been seen to make the no-load current of machines much higher than when they are unsaturated. Therefore, the primary circuit no-load copper loss ($I_o^2 R_1$) neglected by the use of the approximate equivalent circuit can be quite considerable. The required no-load equivalent circuit is as presented in Fig.4., with due temperature and saturation compensation.

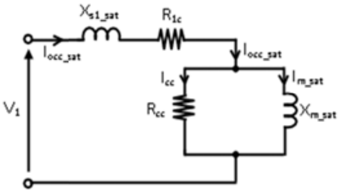

Fig.4: Temperature- and Saturation-Compensated No-load Equivalent Circuit from the Exact T-Equivalent Circuit of Fig.1

In this circumstance, and where there is doubt as to the actual value the primary circuit resistance, R_1, and reactance X_1, typical examples in [4], [12] have shown that close enough values can be obtained by halving the total transformer series resistances and reactance's in each case realized from the laboratory short-circuit test carried out by the reduced voltage method. Thus,

$R_{1c} = \frac{R_{scc}}{2}$ [temperature-corrected primary winding resistance only] (28a)

$X_{s1} = \frac{X_s}{2}$; and $X_{s1\_sat} = \frac{X_{s1}}{K_{sat}} = \frac{X_s}{(2K_{sat})}$ [primary unsaturated and saturated reactance's] (28b)

$Z_{ocs\_sat} = R_{1c} + jX_{s1\_sat}$ [no-load series compensated impedance] (29a)

$$Y_{ocp\_sat} = \left(\frac{1}{R_{cc}}\right) - j\left(\frac{1}{X_{m\_sat}}\right); = \left\{\left[\left(\frac{1}{R_{cc}}\right)^2 + \left(\frac{1}{X_{m\_sat}}\right)^2\right]^{½}\right\} \angle(-\theta_{ocp\_sat})$$

[admittance of the compensated parallel branch circuit and the phase angle] (29b)

$$Z_{ocp\_sat} = \frac{1}{Y_{ocp\_sat}} = \left(\frac{1}{Y_{ocp\_sat}}\right)\angle\theta_{ocp\_sat}$$

$$= [\left(\frac{1}{Y_{ocp\_sat}}\right)cos\theta_{ocp\_sat}] + j[\left(\frac{1}{Y_{ocp\_sat}}\right)sin\theta_{ocp\_sat}]$$

$=$ $Z_{ocp\_sat}\angle\theta_{ocp\_sat}$ [the total parallel branch impedance] (29c)

$$Z_{occ\_sat} = \left[R_{1c} + \left(\frac{1}{Y_{ocp\_sat}}\right)cos\theta_{ocp\_sat}\right] + j\left[X_{s1\_sat} + \left(\frac{1}{Y_{ocp\_sat}}\right)sin\theta_{ocp\_sat}\right];$$

Let

$Z_{occ\_sat\_ac} = \left[R_{1c} + \left(\frac{1}{Y_{ocp\_sat}}\right)cos\theta_{ocp\_sat}\right]$ [active component of Z_{occ_sat}]

and $Z_{occ\_sat\_re} = \left[X_{s1\_sat} + \left(\frac{1}{Y_{ocp\_sat}}\right)sin\theta_{ocp\_sat}\right]$ [reactive aspect of Z_{occ_sat}]

$Z_{occ\_sat} = \left\{\left[\left(Z_{occ\_sat\_ac}\right)^2 + \left(Z_{occ\_sat\_re}\right)^2\right]^{½}\right\}\angle\theta_{occ\_sat}$; where

$$\theta_{occ\_sat} = tan^{-1}\left[\frac{(Z_{occ\_sat\_re})}{(Z_{occ\_sat\_ac})}\right]$$

{total impedance of the compensated exact no-load equivalent circuit and angle} (29d)

$$I_{occ\_sat} = \frac{V_1}{Z_{occ\_sat} =}\frac{(V_1\angle 0)}{(Z_{occ\_sat}}\angle\theta_{occ\_sat}) = \left(\frac{V_1}{Z_{occ\_sat}}\right)\angle(-\theta_{occ\_sat}) = I_{occ\_sat}\angle(-\theta_{occ\_sat})$$

[the no-load current under saturation] (30a)

$pf\__{occ\_sat} = cos(\theta_{occ\_sat})$ [no-load power factor under saturation] (30b)

$P_{occ\_sat} = V_1 I_{occ\_sat} cos\phi_{occ\_sat}$ [total no-load losses under saturation] (31)

(i) *From Short-Circuit Equivalent Circuit and Relevant Experimental Data –*

The equivalent circuit of Fig.5 is appropriate for short-circuit computations under saturation, where

$X_{s\_sat} = \frac{X_s}{K_{sat}}$ [saturated total series reactance] (32a)

and R_{scc} [total temperature-corrected series resistance, as in eqn. (28a)] Here, we have chiefly

$Z_{s\_sat} = R_{scc} + jX_{s\_sat} = \left[\left(R_{scc}^2 + X_{s\_sat}^2\right)^{½}\right]\angle\theta_{s\_sat} = Z_{s\_sat}\angle\theta_{s\_sat}$ (32b)

where $\theta_{s\_sat} = tan^{-1}\left(\frac{X_{s\_sat}}{R_{scc}}\right)$ (32c)

[being the total short-circuit impedance under saturation and the impedance angle]

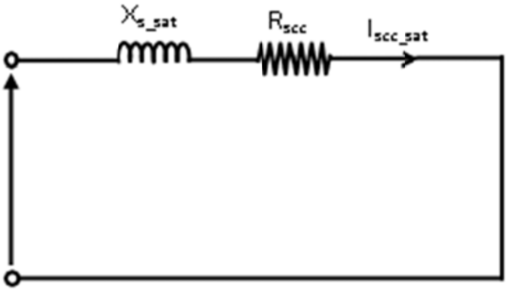

Fig.5: Temperature-Compensated Short-Circuit Equivalent Circuit of Transformer under Saturation and with Quantities referred to the Primary.

$$I_{s\_full} = \frac{V_1}{Z_{s\_sat}} = \frac{(V_1\angle 0)}{(Z_{s\_sat}}\angle\theta_{s\_sat})\ \frac{(V_1}{Z_{s\_sat})}\angle(-\theta_{s\_sat})$$

[short-circuit current on full voltage reflecting saturation] (33)

(iii) From the Complete Exact Equivalent Circuit on Full-Load –

The equivalent circuit for generation of the relevant equations is as provided in Fig.6.

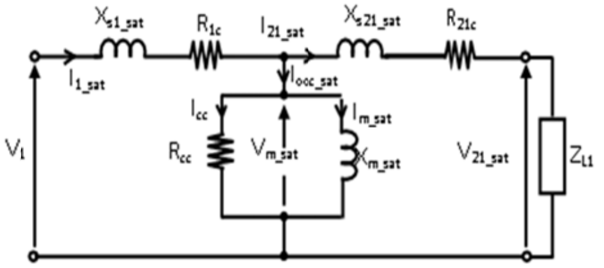

Fig.6: Temperature-Compensated Exact T-Equivalent Circuit of the Transformer with secondary quantities referred to the primary under Saturation Effect.

As earlier explained, it is acceptable to state that

$R_{1c} = R_{21c} = \frac{R_{scc}}{2\ and} X_{s1\_sat} = X_{s21\_sat} = \frac{X_{s\_sat}}{2}$ (34)

The mid-point voltage of the equivalent circuit is given as

$$V_{m\_sat} = I_{occ\_sat} Z_{ocp\_sat} = [I_{occ\_sat}\angle(-\theta_{occ\_sat})][Z_{ocp\_sat}\angle\theta_{ocp\_sat}]$$

$= [I_{occ\_sat}Z_{ocp\_sat}]\angle(\theta_{ocp\_sat} - \theta_{occ\_sat})$; $\theta_{vm\_sat} = (\theta_{ocp\_sat} - \theta_{occ\_sat})$;
$= \frac{[I_{occ\_sat}}{Y_{ocp\_sat}]}\angle\theta_{vm\_sat} = V_{m\_sat}\angle\theta_{vm\_sat}$ (35)

Since the load impedance, $Z_{L1}\angle\theta_{L1}$, remains the same irrespective of the saturation or otherwise of the transformer, the system input impedance shall reflect this fact when computed as follows.

$Z_{in\_sat} = Z_{s1\_sat} + Z_{ocp\_sat}//(Z_{s21\_sat} + Z_{L1})$ (36a)

Let

$Z_{sL1\_sat} = Z_{s21\_sat} + Z_{L1}$
$= Z_{s21\_sat}\angle\theta_{s21\_sat} + Z_{L1}\angle\theta_{L1}$
$= \left(Z_{s21\_sat}cos\phi_{s21\_sat} + Z_{L1}cos\theta_{L1}\right)$

$+j\left(Z_{s21\_sat}sin\phi_{s21\_sat} + Z_{L1}sin\theta_{L1}\right)$

Let $Z_{sL1\_sat\_ac} = \left(Z_{s21\_sat}cos\phi_{s21\_sat} + Z_{L1}cos\theta_{L1}\right)$ [active part of Z_{sL1_sat}]

and $Z_{sL1\_sat\_re} = \left(Z_{s21\_sat}sin\phi_{s21\_sat} + Z_{L1}sin\theta_{L1}\right)$ [reactive part]

so that we have

$Z_{sL1\_sat} = \left\{[\left(Z_{sL1\_sat\_ac}\right)^2 + \left(Z_{sL1\_sat\_re}\right)^2]^{½}\right\}\angle(\theta_{sL1\_sat}) = Z_{sL1\_sat}\angle\theta_{sL1\_sat}$ (38b)

$\theta_{sL1\_sat} = tan^{-1}\frac{[Z_{sL1\_sat\_re}}{Z_{sL1\_sat\_ac}}]$ [phase angle of Z_{sL1_sat}] (38c)

Let $Z_{pL1\_sat} = Z_{ocp\_sat} + Z_{sL1\_sat} = Z_{ocp\_sat}\angle\theta_{ocp\_sat} + Z_{sL1\_sat}\angle\theta_{sL1\_sat}$ (39a)
$= \left(Z_{ocp\_sat}cos\phi_{ocp\_sat} + Z_{sL1\_sat}cos\theta_{sL1\_sat}\right)$
$+j\left(Z_{ocp\_sat}sin\phi_{ocp\_sat} + Z_{sL1\_sat}sin\theta_{sL1\_sat}\right)$

Let $Z_{pL1\_sat\_ac} = \left(Z_{ocp\_sat}cos\phi_{ocp\_sat} + Z_{sL1\_sat}cos\theta_{sL1\_sat}\right)$ [active part of Z_{pL1_sat}]

and $Z_{pL1\_sat\_re} = \left(Z_{ocp\_sat}sin\phi_{ocp\_sat} + Z_{sL1\_sat}sin\theta_{sL1\_sat}\right)$ [reactive part]

so that we have

$Z_{pL1\_sat} = \left\{[\left(Z_{pL1\_sat\_ac}\right)^2 + \left(Z_{pL1\_sat\_re}\right)^2]^{½}\right\}\angle(\theta_{pL1\_sat}) = Z_{pL1\_sat}\angle\theta_{pL1\_sat}$ (39b)

$\theta_{pL1\_sat} = tan^{-1}\frac{[Z_{pL1\_sat\_re}}{Z_{pL1\_sat\_ac}}]$ [phase angle of Z_{pL1_sat}]

$Z_{spL\_sat} = Z_{ocp\_sat}//Z_{sL1\_sat} = \frac{(Z_{ocp\_sat}*Z_{sL1\_sat})}{Z_{pL1\_sat}}$ (40a)

$= \left[\frac{(Z_{ocp\_sat}*Z_{sL1\_sat})}{Z_{pL1\_sat}}\right]\angle(\theta_{ocp\_sat} + \theta_{sL1\_sat} - \theta_{pL1\_sat})$
$= Z_{spL\_sat}\angle\theta_{spL\_sat}$; $\theta_{spL\_sat} = (\theta_{ocp\_sat} + \theta_{sL1\_sat} - \theta_{pL1\_sat})$ (40b)

$Z_{in\_sat} = Z_{s1\_sat} + Z_{spL\_sat} = Z_{s1\_sat}\angle\theta_{s1\_sat} + Z_{spL\_sat}\angle\theta_{spL\_sat}$ (41a)
$= \left(Z_{s1\_sat}cos\phi_{s1\_sat} + Z_{spL\_sat}cos\theta_{spL\_sat}\right)$
$+j\left(Z_{s1\_sat}sin\phi_{s1\_sat} + Z_{spL\_sat}sin\theta_{spL\_sat}\right)$

Let $Z_{in\_sat\_ac} = \left(Z_{s1\_sat}cos\phi_{s1\_sat} + Z_{spL\_sat}cos\theta_{spL\_sat}\right)$ [active part of Z_{in_sat}]

and $Z_{in\_sat\_re} = \left(Z_{s1\_sat}sin\phi_{s1\_sat} + Z_{spL\_sat}sin\theta_{spL\_sat}\right)$ [reactive part]

so that we have

$Z_{in\_sat} = \left\{[\left(Z_{in\_sat\_ac}\right)^2 + \left(Z_{in\_sat\_re}\right)^2]^{½}\right\}\angle(\theta_{in\_sat}) = Z_{in\_sat}\angle\theta_{in\_sat}$ (41b)

$\theta_{in\_sat} = \theta_{1\_sat} = tan^{-1}\frac{[Z_{in\_sat\_re}}{Z_{in\_sat\_ac}}]$ [phase angle of Z_{in_sat}] (41c)

And being that saturation is surveyed relative to the primary terminal voltage, V1, we shall have the input current as

$I_{1\_sat} = \frac{V_1}{Z_{in}=}\frac{(V_1\angle 0)}{(Z_{in\_sat}\angle\theta_{1\_sat})} = \frac{(V_1}{Z_{in\_sat})\angle(-\theta_{1\_sat})} = I_{1\_sat}\angle(-\theta_{1\_sat})$ (42)

Input or primary side power factor under saturation becomes

$pf_{1\_sat} = \left(cos\phi_{1\_sat}\right)$ (43)

The load current is then given as

$I_{21\_sat} = I_{1\_sat} - I_{occ\_sat} = I_{1\_sat}\angle(-\theta_{1\_sat}) - I_{occ\_sat}\angle(-\theta_{occ\_sat})$ (44a)$=$
$(I_{1\_sat}cos\theta_{1\_sat} - I_{occ\_sat}cos\theta_{occ\_sat})$
$-j(I_{1\_sat}sin\theta_{1\_sat} - I_{occ\_sat}sin\theta_{occ\_sat})$

Let

$I_{21\_sat\_ac} = (I_{1\_sat}cos\theta_{1\_sat} - I_{occ\_sat}cos\theta_{occ\_sat})$ [active I_{21_sat}]

and

$I_{21\_sat\_re} = (I_{1\_sat}sin\theta_{1\_sat} - I_{occ\_sat}sin\theta_{occ\_sat})$ [reactive part]

So that we can write

$$I_{21\_sat} = \left\{\left[\left(I_{21\_sat\_ac}\right)^2 + \left(I_{21\_sat\_re}\right)^2\right]^{½}\right\}\angle(-\phi_{21\_sat}) = I_{21\_sat}\angle(-\phi_{21\_sat}) \quad (44b)$$

where $\phi_{21\_sat} = \left[\frac{I_{21\_sat\_re}}{I_{21\_sat\_ac}}\right]$ {ph. angle of I_{21} under saturation} (44c)

Actual load power factor under saturation will be

$$pf_{21\_sat} = \left(cos\phi_{21\_sat}\right) \quad (45)$$

The output or secondary voltage referred to primary

$$V_{21\_sat} = I_{21\_sat}Z_{L1} = \left[I_{21\_sat}\angle(-\phi_{21\_sat})\right]Z_{L1}\angle\theta_{L1} \quad (46a)$$

$$= I_{21\_sat}Z_{L1}\angle(\theta_{L1} - \phi_{21\_sat}) = V_{21\_sat}\angle\delta_{\_sat}$$

$$\delta_{\_sat} = (\theta_{L1} - \phi_{21\_sat})$$

[transmission or displacement angle] (46b)

$$P_{Loss\_sat} = P_{occ\_sat} + P_{co\_sat} = V_1 I_{occ\_sat}cos\phi_{occ\_sat} + I_{21}^2 R_{scc} \quad (47)$$

The apparent power deliverable by transformer on full load under saturation is

$$S_{out\_sat} = V_{21\_sat}I_{21} \quad (48)$$

Active power deliverable by transformer on full load under saturation

$$P_{out\_sat} = S_{out\_sat}pf_{21\_sat} \quad (49)$$

The efficiency of the transformer under saturation is then expressed as

$$eff_{sat} = \left[\frac{P_{out\_sat}}{(P_{out\_sat}+P_{Loss\_sat})}\right]x100 \quad (50)$$

Also, maximum efficiency will take place when P_{co_sat} equals P_{occ_sat}.

Hence, we can write

$$eff_{max\_sat} = \left[\frac{P_{out\_sat}}{(P_{out\_sat}+2P_{occ\_sat})}\right]x100$$

Apparent power at which maximum efficiency occurs under saturation

$$S_{\eta(max)\_sat} = S_{out\_sat}\left[\left(\frac{P_{occ\_sat}}{P_{co\_sat}}\right)\right]^{½} \quad (51)$$

Voltage regulation under saturation is given by

$$V_{reg\_sat} \cong \left\{I_{21\_sat}\frac{\left[R_{scc}cos\phi_{21\_sat}+X_{s\_sat}sin\phi_{21\_sat}\right]}{V_{21\_sat}}\right\}x100 \quad (52)$$

EXPERIMENTATION & THE RESULTS

Figure 7 that follows is a pictorial display of one of the laboratory experiments carried out during this work, which had to do with the determination of the open-circuit characteristic (OCC) of the transformer. The other no-load (or open-circuit) and short-circuit tests for parameter survey were similarly conducted using the relevant apparatus. The results obtained were as provided in Tables 1 and 2. They were all carried out on the HV side of the transformer, which is the secondary side being an inverter transformer (or a step-up transformer).

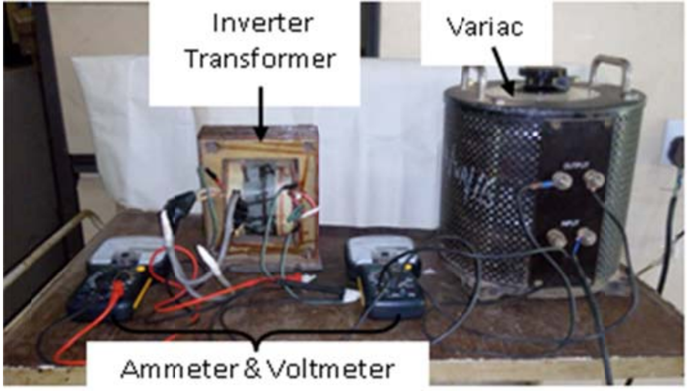

Fig. 7: Picture showing apparatus as set up for Open-Circuit Characteristic Test of the Inverter Transformer.

Table 1: Open-circuit Characteristic Test Result

HV Volts	0V	20V	40V	60V	80V	100 V	120 V	140 V
HV Amps	0A	0.03	0.05 A	0.07 A	0.10 A	0.12 A	0.15 A	0.19 A
HV Volts	160 V	180 V	200 V	220 V	240 V	260 V	280 V	
HV Amps	0.26 A	0.36 A	0.52 A	0.75 A	1.04 A	1.41 A	1.84 A	

Table 2: No-Load and Short-Circuit Tests Results for Parameter Survey

DESCRIPTION OF TEST	APPLIED VOLTAGE	CURRENT DRAWN	POWER CONSUMED	REMARKS
OPEN-CIRCUIT	$V_{\_oc}$ = 230V	$I_{\_oc}$ = 0.86A	P_{oc} = 40W	Winding Temp =35°C
SHORT-CIRCIUT	$V_{\_sc}$ = 20V	$I_{\_sc}$ = 11A	P_{sc} = 160W	

In order to apply the generated equations for performance parameter survey all the current and voltage quantities in Table 2 must first be referred to the primary side, using the following relationships as in [3]:

$$V_{oc} = \frac{V_1}{V_2} * V_{-oc} \ and \ I_{oc} = \frac{V_2}{V_1} * I_{-oc} \ for\ the\ open\ circuit\ test$$

$$V_{sc} = \frac{V_1}{V_2} * V_{-sc} and \ I_{sc} = \frac{V_2}{V_1} * I_{-sc} \ for\ the\ short\ circuit\ test$$

(53)

The values for application are as provided in Table3.

Table 3: No-Load and Short-Circuit Test Results referred to the Primary

OPEN-CIRCUIT TEST	$V_{oc} = 48V$	$I_{oc} = 4.12A$	$P_{oc} = 40W$
SHORT-CIRCIUT TEST	$V_{sc} = 4.17V$	$I_{sc} = 52.7A$	$P_{sc} = 160W$

However, for the magnetization curve (or OCC) determination, the values of applied voltage and exciting current obtained by experimentation were used directly, as there was no need to refer quantities from secondary to primary.

3.0 COMPUTER PROGRAMMING & TEST-RUNNING RESULT

3.1 THE COMPUTER PROGRAMMING

The computer programming was done in MATLAB language. Other programming languages include Foxpro, C/C++, Visual Basic, Pascal, Ada, Fortran and Visual C++ [13]. MATLAB is an acronym for Matrix Laboratory, a product developed and licensed by Math Works Inc. [14]. This is a software package for high performance and visualization, combining capabilities, flexibilities, reliability and powerful graphics, hence, suitable for engineers and scientists. The most important feature of MATLAB is its programming capability, which is relatively easy to learn and to use, and which allows user-developed functions [15].

3.2 PROGRAM TEST-RUNNING RESULT

A) Without Magnetic Saturation

```
Applied_Input_Voltage_in_Volts =    48
Input_Current_of_System_in_Amps =
62.5000
Input_Impedance_of_System_in_Ohms =
0.7680
Input_Power_Factor_Angle_in_Deg =
25.8419
Input_Power_Factor_in_pu =    0.9000
No_Load_Current_in_Amps =    4.0962
No_Load_Power_Factor_in_pu =    0.1724
Short_circuit_Power_Factor_in_pu =
0.7733
System_Actual_Load_Current_in_Amps =
60.1975
Power_Factor_on_Full_Load_in_pu =
0.9227
Load_Power_Factor_Angle_in_Deg =
22.6770
Output_Voltage_on_Full_Load_in_Volts =
43.0917
Voltage_Regulation_in_Percentage =
10.3471
Total_NoLoad_Losses_in_Watts =   33.8983
Total_Load_or_Copper_Losses_in_Watts =
157.1794
Efficiency_on_Full_Load_in_Percentage =
89.7400
Maximum_Efficiency_in_Percentage =
97.2455
Apparent_Power_Delivered_in_VA =
2.5940e+003
Active_Load_Power_Delivered_in_W =
2.3935e+003
Load_for_Maximum_Efficiency_in_VA =
975
Impedance_Voltage_Percentage_Rating =
8.6875
Displacement_Angle_in_Degrees =
1.9647
Load_System_Impedance_in_Ohms =
0.7158
Load_Impedance_Angle_in_Deg =   24.6417
```

The MATLAB plotted magnetizing curve as copied from the Workspace and reduced, is given in Fig. 8a. The line of linearity (LOL), the arrows, the computation and the labeling as provided in Fig. 8b are post-Matlab-plot additions by the author to enable determination of the saturation level, K_{sat}.

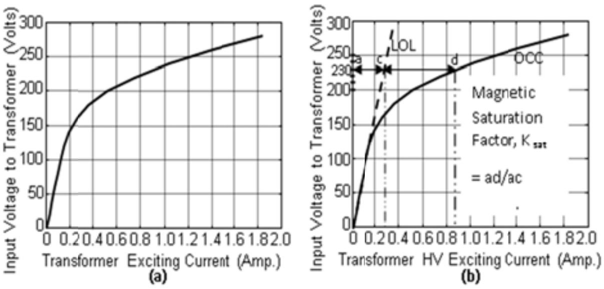

Fig. 8: Open-Circuit Characteristic of the Single-Phase Transformer; **(a)** as Matlab plotted, **(b)** as used to estimate saturation factor

B) With Saturation Effect

```
Applied_Input_Voltage_in_Volts =    48
Input_Current_of_System_in_Amps =
67.2283
Input_Impedance_of_System_in_Ohms =
0.7680
Input_Power_Factor_Angle_in_Deg =
32.1542
Input_Power_Factor_in_pu =    0.8466
No_Load_Current_in_Amps =   12.0913
No_Load_Power_Factor_in_pu =    0.0664
Short_circuit_Power_Factor_in_pu =
0.0830
System_Actual_Load_Current_in_Amps =
60.9190
FullVoltage_ShortCircuit_Current_Amps =
699.8425
Power_Factor_on_Full_Load_in_pu =
0.9211
```

```
Load_Power_Factor_Angle_in_Deg =
22.9095
Output_Voltage_on_Full_Load_in_Volts =
43.6081
Voltage_Regulation_in_Percentage =
9.3841
Total_NoLoad_Losses_in_Watts =   38.5456
Total_Load_or_Copper_Losses_in_Watts =
239.7492
Efficiency_on_Full_Load_in_Percentage =
89.5680
Maximum_Efficiency_in_Percentage =
96.8745
Apparent_Power_Delivered_in_VA =
2.5940e+003
Active_Load_Power_Delivered_in_W =
2.3894e+003
Load_for_Maximum_Efficiency_in_VA =
1040
Impedance_Voltage_Percentage_Rating =
9.1497
Displacement_Angle_in_Degrees =
1.7322
Load_System_Impedance_in_Ohms =
0.7158
Load_Impedance_Angle_in_Deg =   24.6417
DONE
```

4.0 DISCUSSION, CONCLUSION & RECOMMENDATION

4.1 DISCUSSION

A tabular approach as often adopted by this authors in respect of the discussion of the performance of electrical apparatus is used here for the refurbished inverter transformer performance detailing as presented in Tables 4(a) and 4(b) that follow.

Table 4(a): Performance Analysis of the Transformer without Saturation

S/N	PARAMETER & OTHER DESCRIPTIONS	VALUE OBTAINED	VALUE EXPECTED	REMARK
	Magnetic System Performance			
1	No-Load Current, I_o	4.096A (6.55%I_{rated})	3–10%I_{rated} as in [16] p.415; [3] p.37	Good
2	No-Load or Fixed Losses (total), P_{ir}	33.9W (1.13%kVA)	0.5–1.0%kVA for small transformer, as in [17] p.134	Fair
	Electric System Performance			
3	Input power factor, pf1	0.90p.u.	0.90p.u. by proper design selection	Good
4	Load or Copper Losses (total), P_{co}	157.18W (5.26%kVA)	1–1.5%kVA for small transformer, as in [17] p.134	Poor
5	Regulation under load, V_{reg}	10.35%	About 4% on average as in [5] p.26	Poor
6	Impedance Voltage, $V_{z\%}$	8.69%V_{rated}	5–10%V_{rated} as in [3] p.39; 5–17%V_{rated} as in [16] p.416	Good
	Power Transfer Performance Proper			
7	Output power factor, pf21	0.9227p.u.	≥0.90	Good
8	Real Power Delivery	2395.5W (79.83%kVA)	≥2760W (for 0.92 output power factor) i.e. 92%kVA	Good
9	Efficiency (nominal)	89.74% on full load	96–99% as in [3] p. 48	Fair
10	Displacement or Transmission Angle	1.965°	2.8 – 20° as typical with transmission lines [19], [20]	Good

Table 4(b): Performance Analysis of the Transformer under Saturation

S/N	PARAMETER & OTHER DESCRIPTIONS	VALUE OBTAINED	VALUE WITHOUT SATURATION	REMARK
	Magnetic System Performance			
1	No-Load Current, I_o	12.091A (19.35%I_{rated})	4.096A (6.55%I_{rated})	**Very high increase in I_o**
2	No-Load or Fixed Losses (total), P_{ir} or P_{occ}	38.55W (1.29%kVA)	33.9W (1.13%kVA)	***Small increase in P_{ir} or P_{occ}***
A	Magnetic Saturation Factor or Level	3.0p.u.	Not Applicable	Low sat. (1.0 – 1.11 p.u.) Moderate sat. (1.11 – 2.0 p.u.) High sat. (2.0 – 5.0 p.u.) Extreme sat. > 5.0 p.u. [18]
	Electric System Performance			
3	Input power factor, pf1	0.8466p.u.	0.90p.u.	**Considerable decrease in pf1**
4	Copper Losses (total), P_{co}	239.75W (7.99%kVA)	157.18W (5.26%kVA)	**Considerable increase in P_{co}**
5	Regulation under load, V_{reg}	9.38%	10.35%	***Small decrease in V_{reg}***
6	Impedance Voltage, $V_{z\%}$	9.15%V_{rated}	8.69%V_{rated}	***Small increase in $V_{z\%}$***
	Power Transfer Performance Proper			
7	Output power factor, pf21	0.9211p.u.	0.9227p.u.	***Virtually the same***
8	Real Power Delivery, P_{out}	2389.4W (79.65%kVA)	2395.5W (79.83%kVA)	***Small decrease in P_{out}***
9	Efficiency (nominal), eff.	89.57% on full load	89.74% on full load	***Small decrease in eff.***
10	Transmission Angle, δ	1.732°	1.965°	***Small decrease in δ***

From the OCC, it is clear that the transformer begins to get saturated around 150V. At the rated voltage of 230V (on the secondary) the apparatus is seen to operate at a saturation level of 3.0 p.u., (which is a fairly high level). Significantly, under saturation the transformer experiences impoverishment of its input power factor, resulting in a higher reactive power demand for virtually the same active power delivery. A higher input current thus comes into play, being fueled mostly by the 295% increase in the no-load current (for a 3.0 p.u. level of saturation). Load losses increase by 52.53%, and the no-load losses increase by 13.71%. However, there is a small improvement in voltage regulation occasioned by the small improvement in the output voltage due to saturation, whilst the output real power, efficiency, and output power factor remain virtually the same.

4.2 CONCLUSION & RECOMMENDATION

Comparing the values of the transformer exciting current, power losses, active power delivery, voltage regulation and efficiency obtained from this survey (as key performance parameters) with standard performance values as sourced from standard text, it is clear that the performance of the refurbished inverter transformer is fairly good. It is equally conclusive that saturation (if not on the extreme side) does favour the performance of a transformer in many ways, including higher output voltage, lower voltage regulation, higher VA capability at maximum efficiency and lower transmission angle (the latter which makes for higher stability in the transfer of active power).

The evaluation exercise was made a lot easier, quicker and surer by means of the computer software application approach which was adopted and pursued. It is therefore recommendable to transformer repair workshops for easy determination of transformer performance status both for industrial, commercial and educational applications.

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