In Depth Mathematical Algorithm For Heart Rate Variability Analysis

Andris Buikis (Corresponding author), Alberts Aldersons
Institute of Mathematics and Computer Science, Raina bulv. 29, LV1459, Riga, Latvia
buikis@latnet.lv, aldalb@inbox.lv

Abstract. The paper is destined for use in medicine, psychology, in man’s self-development training, breathing technique’s training, in the field of stress resistance, health promotion, strengthening of the capacity for work; and it relates to the apparatus and methods for detection of the heart rate variability and it’s may be used in providing biofeedback during training sessions of organism’s vegetative balance and coherence. We involve in our algorithm the regulation and adaption in the form of histogram. Our in-deep mathematical algorithm understands more deeply the details of many aspects of the adaptation process.

Keywords—Heart; Heart rate variability; Biofeedback; Method for detection; Stress resistance; Coherence; Specific determinant fragments; Histogram.

I. Introduction

Heart Rate Variability (HRV) has occupied considerable and leading state as a non-invasive method for investigation of the activity levels and dynamics of interaction of the sympathetic and parasympathetic branches of the Vegetative Nervous System (VNS). During last two decades investigation of the physiological mechanism of HRV, development of new apparatus and new computer programs for its registration, interpretation and practical utilization are considerably progressed.

In our paper [1] and patent in Latvia [2] we proposed quick mathematical model and algorithm for HRV. In this paper we develop this algorithm in form of specific determinant fragment and histogram. In the first part we in the short form review our algorithm from [1].

II. Formulation of Problem and Nowadays Statement

Contrary to the view that under optimal conditions the heart beats sequence should remind the metronome, this is definitely not so [3]. Due to influence of Autonomic Nervous System, affecting the sinus node (nervous center, located in the heart, which activates each of the next cardiac cycle starting after a pause), pulse beats followed each other at different time intervals, and, as a result, the time span between two consecutive heart beats can vary over a wide range – from 400 to 1500 msec. Plotting these following time intervals graphically, we get a wavy line. It is called the Heart Rate Variability (HRV) line. It turned out that this curve is very informative [3].

When registering a heartbeat, we get a simple series of numbers (intervals between every two consecutive heartbeats in millisecond’s, in average 70 numbers during minute, such as, for example, 721, 753, 835, 802, 799, etc.) from which we may derive many different indicators of the activity of vegetative nervous system, and, in addition, these numbers are each characterized by strongly different physiological or psychological conditions. That is why; we see a rapid increase of searches of new algorithms, approbation of new mathematical models.

Any technique, that allows you to record an electrocardiogram, is valid. As recording equipment the conditions does not play important role (as it for many other so called psycho physiological methods, for example, galvanic skin response), the technical details of the HRV record is no longer even object for serious discussions in the scientific literature.

We should start with the fact that the HRV was one of the chief methods used for evaluation of physiological state in aerospace medicine and psychology (it was in the period around years 1950-1980, mainly in Russia). There are many studies that indicate the relationship between emotions and changing SRV indicators. HRV may be used as an indicator of risk prediction after myocardial infarction [3].

Some ideas about mathematical aspects of the SRV methodology and measurements standards: the variations in heart rate may be evaluated by a number of methods. Perhaps the simplest to perform are the time domain measures. In these methods, either the heart rate at any point in time or the intervals between successive normal complexes are determined. In continuous ECG record, each QRS [3] complex is detected, and the so-called normal-to-normal (NN) intervals (that is, all intervals between adjacent QRS complexes resulting from sinus node depolarization’s) or in the instantaneous heart rate is determined. The simplest variable to calculate is the standard deviation of the NN intervals (SDNN), that is, is square root of variance. Other commonly used statistical variables calculated from segments of the total monitoring period include SDANN [3], the standard deviation of the average NN intervals calculated over short periods, usually 5 minutes, which is an estimate of the changes in heart rate due to cycles longer than 5
The basic disadvantage of parametric methods is the need of verification of the suitability of the chosen model and of its complexity (that is, the order of the model).

The most commonly used measures derived from interval differences include RMSSD, the square root of the mean squared differences of successive NN intervals, NN50, the number of interval differences of successive NN intervals greater than 50 ms (milliseconds), and pNN 50, the proportion derived by dividing NN50 by the total number of NN intervals. All of these measurements of short-term variation estimate high-frequency variations in heart rate and thus are highly correlated. Since many of the measures correlate closely with others, the following four measures are recommended for time domain HRV assessment:

1. SDNN (estimate of overall HRV),
2. HRV triangular index (estimate of overall HRV),
3. SDANN (estimate of long-term components HRV),
4. RMSSD (estimate of short-term components HRV).

Shortly about Frequency Domain Methods. The analysis of the tachogram has been applied since the late 1960s. Power spectral density (PSD) analysis provides the basic information of how power (variance) distributes as a function of frequency. Independently of the method used, only an estimate of the true PSD of the signal can be obtained by proper mathematical algorithms.

Methods for the calculation of PSD may be generally classified as nonparametric and parametric. In most instances, both methods provide comparable results. The advantages of the nonparametric methods are:

1. the simplicity of the algorithm used (fast Fourier transform(FFT) in most of the cases);
2. the high processing speed.

While the advantages of parametric methods are:

1. smoother spectral components that can be distinguished independently from preselected frequency bands;
2. easy post processing of the spectrum with the an automatic calculation of low- and high-frequency power components with identification of the central frequency of each component;
3. an accurate estimation of PSD even on a small number of samples on which the signal is supposed to maintain stationary.

The invention relates to breathing technique training, stress reduction, health and capacity building areas, in particular, to apparatus and methodologies for determining heart rate variability and its application in biofeedback of various vegetative body’s balance and consistency conditions during training.

It is well known that the autonomic nervous system (ANS) regulatory mechanisms are affecting many organs and systems of human body. The system works as a very complex physiological oscillator unit. One of the main functions of ANS is provision of the optimal balance between the oscillators. Any protracted regulatory balance disorder may lead to functional disturbances that can damage the human body. One of the main creators of this imbalance is emotional stress. There are many methodologies that apply for funds to reduce emotional stress, its positive impact on ANS regulatory function. This technology is designed to switch the ANS to specific “resonant” state, characterized by many physiological parameters synchronous oscillator, coherent, sine wave-like characteristic, such as heart rate (SD), BP (blood pressure), respiration rate (ER), and so on.

Good representing of resonant states is HRV, or respiratory sinus arrhythmia where the heart rhythm...
(HR) oscillates synchronously with the respiratory cycle. Such synchronization automatically regulates many other body functions, including certain brain functions, including certain brain rhythms and metabolic processes. It is also known that the HRV is the representation not only of his rhythm, but also of the size of the state of harmony of many other autonomic nervous system reactions, so that could be used as the body’s overall coherence and the coincidence index. Since the aim of the stress-reduction techniques is to create a resonance between the oscillating physiological parameters, it is important to be able to assess the dynamics of this resonance significance and stability in quantitative values, and generate feedback to reinforce positive changes.

To make SRV analysis and evaluation of the SRA parameters electrocardiographic (ECG), signal is commonly used. Inter-beat intervals are derived from the ECG as the intervals between two adjacent R-waves. It is very accurate and promising method, although quite inconvenient and relatively expensive. Photoplethysmography is used as an alternative method, applying the small finger position sensor [4], [5]. The sensor emits infrared light into the skin. The emitted light in part, linked by blood flow. Light absorption/reflection coefficient is proportional to blood flow changes. Pletizmogram signal contains periodic rapid elevations showing vascular pulsations. They can be used to determine heart inter-beat intervals, characterized by the distances between two pletizmogramas peaks. In all we know the source of information where the feedback is used for realization of HRV, the calculation algorithm is characterized by any of the similar characteristics – the time interval, necessary for heart rate records to allow quality and standards of appropriate HRV calculations. In classical case, by international agreement, this amount of time should be 5 minutes long [3]. After the prototype and analogues, the most modest number of pulse beats for, calculating the approximate SRV data must be at least100 pulse beats (1950-up to 80-year Soviet cosmonaut psycho physiological monitoring of the condition). Absolutely the smallest number of pulse beats needed to make something quite inaccurate, but calculate the average heart rate, pulse rate is 10-15 beats, but the average pulse is not the pulse rate variability.

This means that when working on a prototype or analogue techniques:

a) To collect data before the feedback signal for generation we must wait at least 1-6 minutes;

b) Each subsequent result can be obtained again only after 1-6 minutes;

c) The work in “moving average” mode is in better shape, because the waiting for “dead” period is necessary only in the beginning, and then we are able to calculate for each pulse all SRV figures for the previous 1.5-6 minute period, but the improvement is somewhat apparent (illusory), because in essence we do not obtain a HRV description of each concrete short time moment, but we have the result of previous period average.

All these methods are satisfactory, if we are interested in slow physiological responses, which are changing in tens of minutes and hours. But usually all the physiological and psychological processes that we want to adjust to the reverse link are not so long. Even breathing, is used to improve to adjust the pulse variability. Very rarely, in exercises (including yoga), a respiratory cycle lasts longer than one minute. So, with the prototype and its analogues, we only very conditionally can hope that we as a feedback signal we truly use peculiar HRV indicators. But any stress during minutes may generate enormous changes, and that is why adequate HRV record figures are essential.

3.2. Practical application of the proposal in detail

The first part is exactly the same for all pulse variability detecting methods. It is as follows. By means of any standard EKG or pulse beat recording device time intervals between each successive electrocardiogram QRS complex or pulse wave must be fixed with the an accuracy of at least 1 ms (milliseconds).

But the momentary heart cycle length (or instantaneous pulse) rates in practice are not used as pulse feedback indicators. They are very changeable, and physiological benefits lies precisely in its ability to characterize this variability mathematically, i.e., to find an algorithm that is the best and most practical to characterize a specific physiological conditional or situation, and can be used as feedback indicators. Standard HRV indicators are appropriate for scientific purposes, but are less adequate for scientific purposes as biofeedback elements. Our work allowed us to see the possibility to derive a special mathematical algorithm for processing the instantaneous pulse data with the aim to generate a realistic indication of the vegetative n. s. regulatory processes, and, thus, create a parameter, which could serve as a convenient indicator used for biofeedback.

Usually calculations that are offered and used for biofeedback signal need at least 100-500 test points, located before the instantaneous time point recorded. For realization of our algorithm only 2 points before the instantaneous time point recorded are necessary, and, more importantly, the conclusions we make with only one, the most recent time interval, with his relationship to the previous interval, which is just a report. The result therefore does not describe the average physiological state of several minutes, but about 1 second. Thus, feedback can be shown immediately after the fourth heartbeat.
3.3. Our algorithm is implemented as follows.

The following convention is adopted:

\( P(n) \) = the time moment of the current heart beat (fourth, if the calculations carried out at the fourth heart percussion);

\( P(n-1) \) = third, if the calculations carried out at the fourth heart percussion;

\( P(n-2) \) = second, if the calculations carried out at the fourth heart percussion;

\( P(n-3) \) = first, if the calculations carried out at the fourth heart percussion;

\( T(n) \) = time interval between \( P(n) \) and \( P(n-1) \);

\( T(n-1) \) = time interval between \( P(n-1) \) and \( P(n-2) \);

\( T(n-2) \) = time interval between \( P(n-2) \) and \( P(n-3) \).

Beginning from 4-th pulse beat and forth, each pulse beat is granted with following designation (Parameter \( V(x) = + \) or \( V(x) = - \)) according to following algorithm:

\( V(n) = +^*, \) if \( T(n) > T(n-1) \);

\( V(n) = -^*, \) if \( T(n) < T(n-1) \);

\( V(n) = 0^*, \) if \( T(n) = T(n-1) \);

\( V(n-1) = +^*, \) if \( T(n-1) > T(n-2) \);

\( V(n-1) = -^*, \) if \( T(n-1) < T(n-2) \);

\( V(n-1) = 0^*, \) if \( T(n-1) = T(n-2) \).

On each pulse beat beginning from the fourth beat, the following calculations are made:

If \( V(n) = +^* \) and \( V(n) = +^* \), then PBFS (Positive Biofeedback Signal) = PBNS + 1;

if \( V(n) = -^* \) and \( V(n) = -^* \), then PBFS (Positive Biofeedback Signal) = PBNS + 1;

if \( V(n) = +^* \) and \( V(n-1) = -^* \), then NBFS (Negative Biofeedback Signal) = NBNS + 1;

if \( V(n) = 0^* \) and \( V(n-1) = +^* \), then NBFS = NBNS + 1;

if \( V(n) = +^* \) and \( V(n-1) = 0^* \), then NBFS = NBNS + 1;

if \( V(n) = -^* \) and \( V(n-1) = -^* \), then NBFS = NBNS + 1;

if \( V(n) = 0^* \) and \( V(n-1) = 0^* \), then NBFS = NBNS + 1.

For time period of 20 sec or longer the “Central Index (CI)” is calculated as following:

\[ \text{"Central Index (CI)"} = \frac{\text{amount of all PBFS/ (all PBFS and NPBS amount)}}{100} \text{ (percent)}. \]

This calculation can be performed in both ways: immediately after each heart beat, or as a retrospective analysis of a particular situation, particular time interval may be made. Since CI is always within the range between 0 and 100%, and the value estimate is unchangeable, it may well be used as comparative indicator and benchmark for the one and the same person at different life, work and health situations, as well as various human condition inter-comparisons. CI is very dynamic, if you use it as a moving average, for example, from 20 heart beats, and, at the same time, it is also a solid, stable indicator, that can be used to compare averages for different people, or one and the same human figures on different days, months or years, if the expense of a 5-minute long recorded is performed in standardized conditions. Many of our measurements allow introducing approximate boundaries of these index-readings above 50% usually mean good health and high performance. Specially trained people, who manage yoga and deep breathing techniques, are able to increase this figure, and long to hold 70-90% range. To emotional stress, the figure falls below 30%, and some situations, it is only 3-6%.

IV. The New Deeper Algorithm

Thus, as follows from the previous one, and making it simpler, from the RR intervals we evaluated and pointed out one characteristic - whether each of the following RR intervals maintains their direction, or the direction changes. Biological sense of such interpretation in our point of view is the following: the longer are the unchangeable direction periods, the better and more beneficial are the circumstances for vegetative nervous system and organism as the whole: better are the possibilities for stabilization and recovery for many processes in human body. Our previous algorithm demonstrates it dramatically – it rest position in comparison to anxiety, of psycho-emotional tension can be reduced 10 and more times.

However, as demonstrated in our future work, our algorithms proved necessary to develop further, because it revealed new opportunities to analyze and understand more deeply the details of many aspects of the adaptation process, particularly emotional stress. We found that this one-way RR interval amount consists of individual accurately determined fragments. This means the following: RR intervals of the debt may be extended or reduced in continuous series – the two, three, four, and so on. Further, we found that this approach opens new opportunities and much broader understanding on the general regulation and the adaptation processes of human body. We found that mathematically the most obvious and most convenient way is to analyze this process of adaptation in the form of histogram. Therefor we created a new mathematical algorithm for investigation of heart rate variation. It is as follows:

```java
ab1ind=0;
ab1sk=0;
vidab1ind=0;
ab1etaps=0;
ab1enkurs=0;
ab1kopsk=0;
for (v=5; v<=z; v++) {
    ...
}
```
ab1kopsk=ab1kopsk+1;
ab1etaps=0;
if (rr[v]>rr[v-1] && rr[v-1]>rr[v-2])
{ab1ind=ab1ind+1; ab1sk=ab1sk+1; ab1etaps=5;}
if (rr[v]<rr[v-1] && rr[v-1]<rr[v-2])
{ab1ind=ab1ind+1; ab1sk=ab1sk+1; ab1etaps=5;}
if (ab1etaps==0){
  hi1=hi1+1;
  if (ab1ind==2){hi2=hi2+1;}
  if (ab1ind==3){hi3=hi3+1;}
  if (ab1ind==4){hi4=hi4+1;}
  if (ab1ind==5){hi5=hi5+1;}
  if (ab1ind==6){hi6=hi6+1;}
  if (ab1ind==7){hi7=hi7+1;}
  if (ab1ind==8){hi8=hi8+1;}
  if (ab1ind==9){hi9=hi9+1;}
  if (ab1ind==10){hi10=hi10+1;}
  if (ab1ind==11){hi11=hi11+1;}
  if (ab1ind==12){hi12=hi12+1;}
  if (ab1ind==13){hi13=hi13+1;}
  if (ab1ind==14){hi14=hi14+1;}
  if (ab1ind==15){hi15=hi15+1;}
  ab1ind=1;
}
vidab1ind=hi1+hi2+hi3+hi4+hi5+hi6+hi7+hi8+hi9+hi10 +hi11+hi12+hi13+hi14+hi15;
hi1=hi1*100/vidab1ind;
hi2=hi2*100/vidab1ind;
hi3=hi3*100/vidab1ind;
hi4=hi4*100/vidab1ind;
hi5=hi5*100/vidab1ind;
hi6=hi6*100/vidab1ind;
hi7=hi7*100/vidab1ind;
hi8=hi8*100/vidab1ind;
hi9=hi9*100/vidab1ind;
hi10=hi10*100/vidab1ind;
hi11=hi11*100/vidab1ind;
hi12=hi12*100/vidab1ind;
hi13=hi13*100/vidab1ind;
hi14=hi14*100/vidab1ind;
hi15=hi15*100/vidab1ind;
println(ab1kopsk);
println(vidab1ind);

This algorithm is the new mathematical model of the paper. This allows you to get a new view of the HRV, dividing the groups by one direction, so called specific determined fragments. Authors understand that this new view is important for medical uses, but the goal of this article is not a directly medical. The new, in-depth mathematical algorithm – it is the task of this article. Authors plan to continue this new direction with other publications.

Following is the presentation the four images, in which the RR intervals in specially determined fragments appear in the form of a histogram.

Fig. 1. Histogram for restless body. Green is our algorithm from section III; paper [1], patent [2]. Light color is deep algorithm. Black is from books [6]–[8].

Fig. 2. Histogram for peaceful body. Green is our algorithm from section III; paper [1], patent [2]. Light color is deep algorithm. Black is from books [6]–[8].
The works of Russian authors in the 1960s and 1970s of previous century are related with the preparation of the spaceman for cosmic flights. Because this reason we added the results of these studies in figures 1-4. A little further in figures in 5 and 6 we compare our in-depth mathematical algorithm with recent interesting Finnish Biosignal Analysis and Studies of Medical Imaging Group.

Fig.3. Histogram for relaxation body. White is our algorithm from section III, paper [1], patent [2]. Light color is new deep algorithm. Black is from books [6]-[8].

Fig.4. Histogram for optimization body. White is our algorithm from section III, paper [1], patent [2]. Light color is new deep algorithm. Black is from books [6]-[8].

However, as our next work showed, our algorithm had to be developed further, because we found out new possibilities to analyze more detailed and understand many details of process of adaptation, especially in circumstances of emotional stress. We found out that the sum of these one directional RR intervals consists of separate particularly determinable fragments. It means that RR intervals can be prolonged in continuous series – two by two, three by three, etc. With great skills, this number can reach 10 and more.

Our small experience shows that with the apparatus on portable computers and other devices allow you to get more than the 8-10 one row values existing heart rate variation. Excluding these devices, manages rhythm length increased to 12-13. It is a very interesting field of research [9], which studies should continue.

Finnish scientists developed the complex, which you can get from authors. This is a serious piece of work, published in [10]-[12]. Recently appeared a new complex of programs, which was developed by Finnish group of scientists, these programs can be obtained freely. We used this complex for data which are described in fig.3, 4 by our algorithm. The results are displayed in fig. 5, 6. As we can see, our new algorithm and Kubios HRV 2.2 give different results. It will be interesting to compare them in future. Our new algorithm is on sub-sensorium level.

Paper [13] describes an algorithm which similarly as our algorithm, shows fast processing HRV results. Interesting paper is [14], very informative and useful.

Fig.5. Analysis the heart rate variability from fig. 3 by program from [10].
Fig. 6. Analysis the heart rate variability from fig. 4 by program from [10].

Conclusions

Training technique to reduce the emotional stress, psychological coherence and automatic balance, through a special breathing movements, body postures, psychological, spiritual and other practices that comply with heart rate variability feedback control, in which the parameter estimate can be made and the feedback signal is generated at each pulse of percussion, the calculations are made for direct-time, and are played on portable devices or PC screen, and the parameters used in various systems management (physical medicine equipment, stress management, training and work on capacity building, light-sound stimulation systems) varies with the fact that, in order to improve the efficiency of the method 1 st than 20 seconds over a longer period, calculate

the “Central Index (CI),” according to the formula: amount of all PASS/(all PASS and NASS amount)*100 (percent);

2nd CI is calculated as a single value for a selected period of time, both as a “rolling average” of at least 5 previous pulse intervals between adjacent pulse beats;

3rd PASS, NASS and CI will be calculated direct-time, and are played on portable devices or a PC screen;

4th PASS, NASS and CI using different management systems (physical medicine equipment, training systems, different light-sound stimulation systems).

Our new extended in-depth algorithm of heat rate allows increasing significantly the accuracy of detection of emotional stress. In addition, it opens up new opportunities for the investigation of stress resistance, adaptation and adjustment process, allowing comprehending person’s specific, individual features and hidden adaptation and regulation capacity backups at rest, as well as at a wide variety of loads - mental, emotional, physical (including thermal). We need extended practical test for the algorithm, as well as specific practical methodologies and development of software. In our view, this could be a very promising human self-development and health improvement training algorithm, which could be applied to individual characteristics of each person.

Acknowledgment

This work has been supported by Latvian Council of Sciences (grant 623/2014).

References


